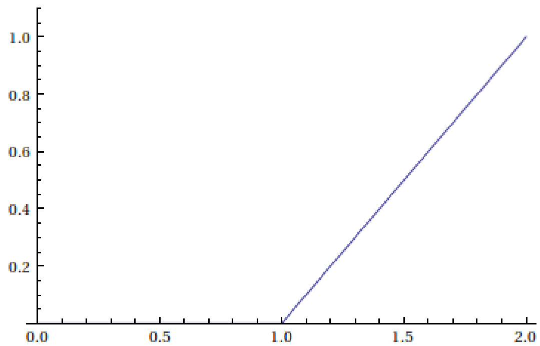
1 Math 1 Optional practice HW list α

1. Let $f(x)$ be the function defined for $0 \le x < 2$ by

$$f(x) = \begin{cases} 0 & \text{if } 0 \le x < 1 \\ x - 1 & \text{if } 1 \le x < 2 \end{cases}$$



In each part, make sure your answer is a function. On the first test, I subtracted points for just calculating Fourier coefficients and not stating a formula for the function as a series.

   (a) Calculate the Fourier sine and cosine series for the 2-periodic function extending f

   (b) Calculate a Fourier sine series for f. (That is: extend f to a 4-periodic odd function, then calculate the Fourier series)

   (c) Calculate a Fourier cosine series for f. (That is: extend f to a 4-periodic even function, then calculate the Fourier series)

2. Let g be the 4-periodic function from problem 1(c), and $h(x)$ be the function in problem 1(b).

   (a) Use the identity $\cos(kx) = \frac{1}{2}e^{ikx} + \frac{1}{2}e^{-ikx}$ to obtain complex exponential Fourier series for $g(x)$ and $h(x)$.

   (b) Calculate a Fourier series for the derivative $g'(x)$.

   (c) Differentiate $h(x)$. Since h has a jump, your answer will include a Dirac mass. Comment: the Fourier series for the distribution $h'(x)$ doesn't converge pointwise. It does however "converge weakly"; we'll talk about this in class.

   (d) Calculate a Fourier series for the integral $\int_0^x h(t)\,dt$.

3. For each of the three functions in problem 1, determine whether the Fourier series converges uniformly. If not, then calculate the overshoot at each point near which the Fourier series doesn't converge uniformly.

4. Suppose a 7-periodic function is absolutely continuous except for jumps of -2 at $x = 3$ and 6 at $x = 4$. Determine the asymptotics of the Fourier coefficients c_k.

5. The wave equation with speed $c = 1000m/s$ is

$$u_{tt} = \left(1000\frac{m}{s}\right)^2 \alpha u_{xx}$$

Let's measure the displacement u and position x in meters, and the time t in seconds. Suppose a string of length $1.5m$ vibrates according to the wave equation, and it's initial position, velocity and boundary conditions are given by

$$\begin{aligned}
u(x,0) =& 0 \\
u_t(x,0) =& \begin{cases} 0.001\frac{1}{s}x & \text{if } 0 < x < 0.75m \\ -0.001\frac{1}{s}(x-0.75m) & \text{if } 0.75m < x < 1.5m \end{cases} \\
u(0,t) =& 0 \\
u(1.75,t) =& 0
\end{aligned}$$

Give a Fourier series solution for the motion of the string.